Hysteresis Oscillators

By Kenneth Low

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Equal to 27

Equal to 27

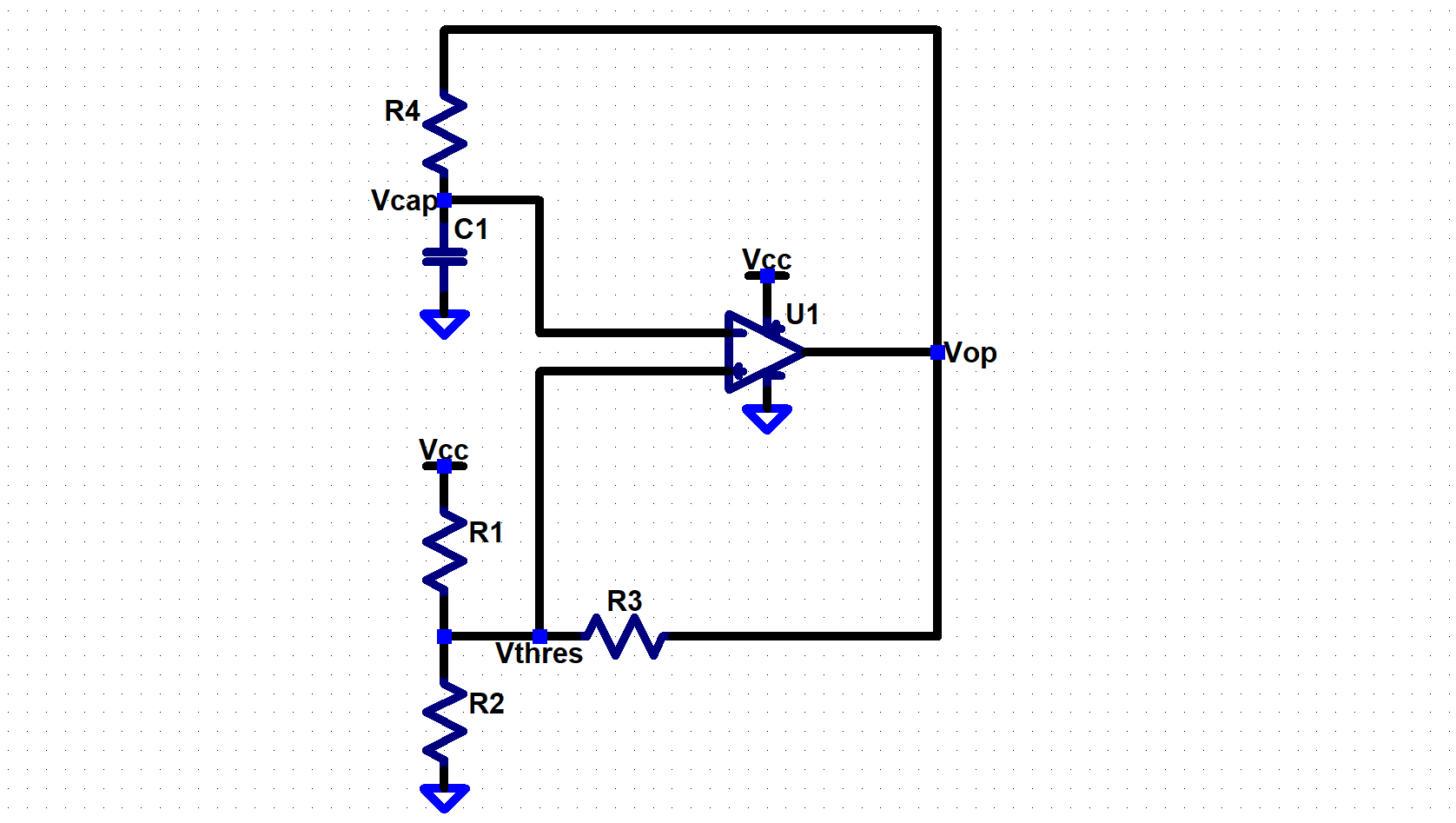


Figure 1: Hysteresis Oscillator Circuit

# Summary

The Hysteresis oscillator, also known as a Schmitt Trigger oscillator, is a type of comparator based relaxation oscillator. The circuit uses an operational amplifier as a comparator. , , and are used to provide hysteresis, positive feedback, which is creates a comparator trigger point that varies with the voltage output of the opamp, . The RC circuit, and , charges or discharges, up to or down to the current trigger point, which is set by . Ultimately, this circuit can produce a square wave output with a frequency set by the RC circuit and comparator trigger points.

# Hysteresis Resistors

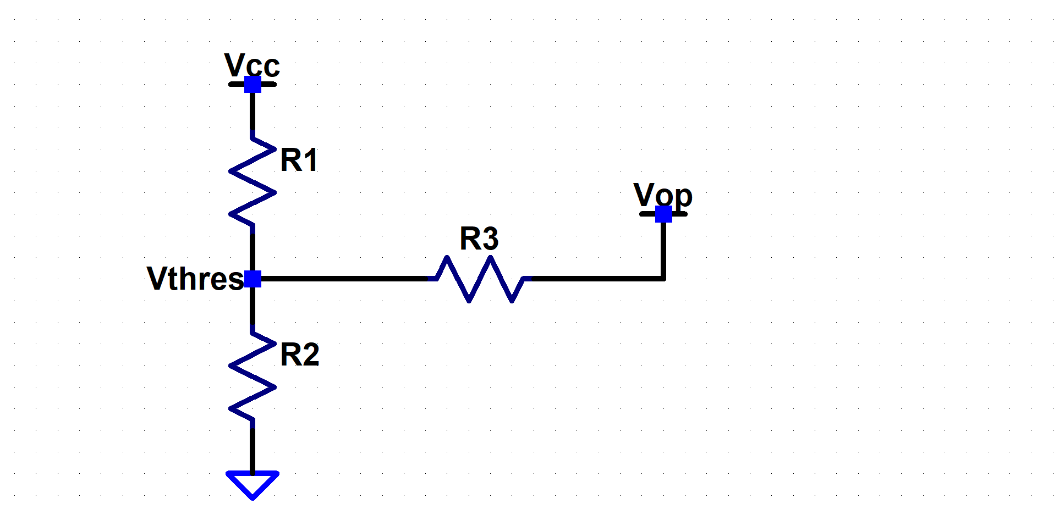


Figure 2: Hysteresis Resistors

The hysteresis resistors are used to produce a set of trigger points, and . This section of this paper explores several possible arrangements to easily realize a set of resistors, , , and , which produce a desired set of trigger points. This is simplified by using an ideal model of opamp operation, the intent is that the ideal case makes finding values in the ball park of what is desired easier.

To produce exact values for the selected resistors use the selected resistor values with the generalized equation and use a specific opamp’s high and low output voltage swing, in place of for the actual and values.

Keep in mind that some threshold voltages close to or may not be realizable due to the opamp output swing not being capable of generating voltages high or low enough. A rail to rail opamp is recommended to achieve the highest flexibility because of the larger voltage swing inherent in the design of said opamp’s.

## Generalized Equation

Using nodal analysis a generalized equation for , the threshold voltage for triggering the comparator response of the hysteresis oscillator, can be found.

Equation 1: General Equation for Hysteresis caused threshold Voltage Values

Where will have a high and low voltage state, producing a and .

## Particular Solutions for the Generalized Equation

To make evaluating a specific set of resistance values, , , and easier a particular solution to the generalized equation can be implemented.

### R1, R2 and R3 set to be equal

One particular solution can be arrived at by setting , and to be equal to :

Equation 2: Particular Solution setting ==

This equation can be further simplified by assuming a set of ideal output states from the opamp[[1]](#footnote-1).

|  |  |
| --- | --- |
| *Ideal Low State* | *Ideal High State* |
|  |  |

*Equation 3: Ideal Low State threshold voltage in proportion to , given*

Equation 4: Ideal High State threshold voltage in proportion to , given

Setting the hysteresis resistors to be equal results in a set of threshold voltage points which are for the lower threshold and for the high threshold.

### R1 and R2 set to be equal

Another particular solution can be arrived at by setting and to be equal to , and setting to be some factor of , times :

Equation 5: Particular Solution setting and

This equation can be further simplified by assuming a set of ideal output states from the opamp.

|  |  |
| --- | --- |
| *Ideal Low State* | *Ideal High State* |
|  |  |

*Equation 6: Ideal Low State threshold voltage in proportion to , given*  and

*Equation 7: Ideal High State threshold voltage in proportion to , given*  and

Simplifying down to this set of equations allows a few assumptions about how adjusting in relation to , affects the threshold voltages of this resistor configuration.

#### Large in Relation to

If is very large in comparison to , this would require to also be very large, which allows us to take the limit of x, from Equation 6 and Equation 7, as it approaches infinity to evaluate for very large values of :

Taking the limit as approaches infinity shows that when and the high and low threshold voltages approach roughly .

#### Small in Relation to

If, though, is much smaller than , meaning x is also very small:

Taking the limit as x approaches zero shows that when and the high and low threshold voltages approach and .

#### Equal to

If is set equal to , meaning :

This result was shown in the above section.

#### Equal to Center Point

One more insight which can be seen from the simplified equations is: e simplified equations is"

This effectively means that the threshold values will be center balanced with at the center of the threshold points.

#### Equal to Threshold Voltage Span vs Value

Taking the difference between and gives us an equation for the span of the threshold voltages as a proportion of :

Graphing the difference function provides insight into how the threshold voltage span, in relation to , changes with different values of .

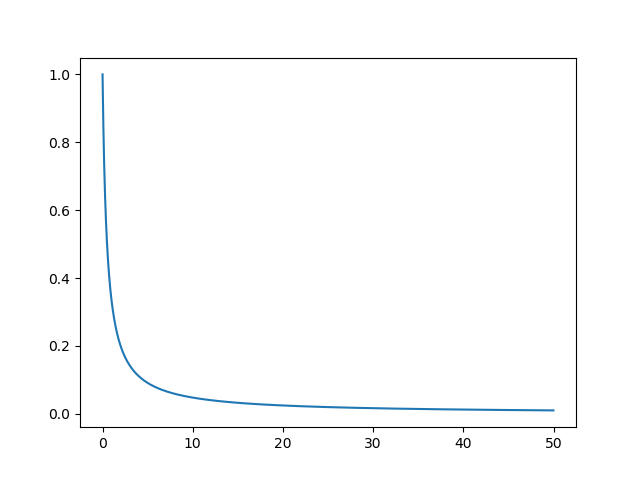


Figure 3: Graph of Threshold Voltage span when up to equal to 50

#### Example 1:

With find which gives a and a .

### R1 and R3 set to be equal

Another particular solution can be arrived at by setting and to be equal to , and setting to be some factor of , times :

Equation 8: Particular Solution setting and

This equation can be further simplified by assuming a set of ideal output states from the opamp.

|  |  |
| --- | --- |
| *Ideal Low State* | *Ideal High State* |
|  |  |

*Equation 9: Ideal Low State threshold voltage in proportion to , given*  and

*Equation 10: Ideal High State threshold voltage in proportion to , given*  and

Simplifying down to this set of equations allows for assumptions to be made about how adjusting in relation to , affects the threshold voltages of this resistor configuration.

#### Large in Relation to

If is very large in comparison to , this would require to also be very large, which allows us to take the limit of x, from Equation 9 and Equation 10, as it approaches infinity to evaluate for very large values of :

Taking the limit as approaches infinity shows that when and the high voltage threshold approaches and the low threshold approaches .

#### Small in Relation to

If, though, is much smaller than , meaning x is also very small:

Taking the limit as x approaches zero shows that when and the high and low threshold voltages approach . This makes sense because is attached to ground. Making smaller would push the potential at closer to ground and making exactly would cause to effectively be shorted to ground.

#### Equal to

If is set equal to , meaning :

This result was also shown in a previous section above.

#### Equal to Threshold Voltage Span vs Value

Taking the difference between and gives us an equation for the span of the threshold voltages as a proportion of : e simplified equations is"

Graphing the difference function provides insight into how the threshold voltage span, in relation to , changes with different values of .

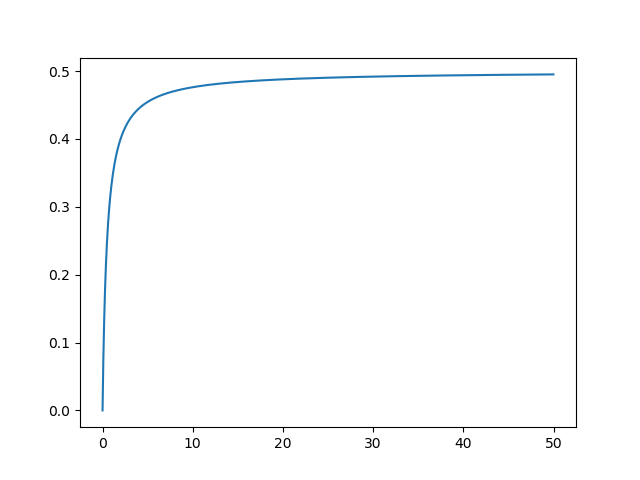


Figure 4: Graph of Threshold Voltage span when up to equal to 50

#### Example 2:

With find and which give a .

### R2 and R3 set to be equal

Another particular solution can be arrived at by setting and to be equal to , and setting to be some factor of , times :

Equation 11: Particular Solution setting and

This equation can be further simplified by assuming a set of ideal output states from the opamp.

|  |  |
| --- | --- |
| *Ideal Low State* | *Ideal High State* |
|  |  |

*Equation 12: Ideal Low State threshold voltage in proportion to , given*  and

*Equation 13: Ideal High State threshold voltage in proportion to , given*  and

Simplifying down to this set of equations allows for assumptions to be made about how adjusting in relation to , affects the threshold voltages of this resistor configuration.

#### Large in Relation to

If is very large in comparison to , this would require to also be very large, which allows us to take the limit of x, from Equation 12 and Equation 13, as it approaches infinity to evaluate for very large values of :

Taking the limit as approaches infinity shows that when and the high voltage threshold approaches and the low threshold approaches 0.

#### Small in Relation to

If, though, is much smaller than , meaning x is also very small:

Taking the limit as x approaches zero shows that when and the high and low threshold voltages approach . This makes sense because is attached to . Making smaller would push the potential at closer to and making exactly would cause to effectively be shorted to .

#### Equal to

If is set equal to , meaning :

This result was also shown in a previous section.

#### Equal to Threshold Voltage Span vs Value

Taking the difference between and gives us an equation for the span of the threshold voltages as a proportion of : e simplified equations is"

This function is the same as it was in the case. Graphing the difference function provides insight into how the threshold voltage span, in relation to , changes with different values of .

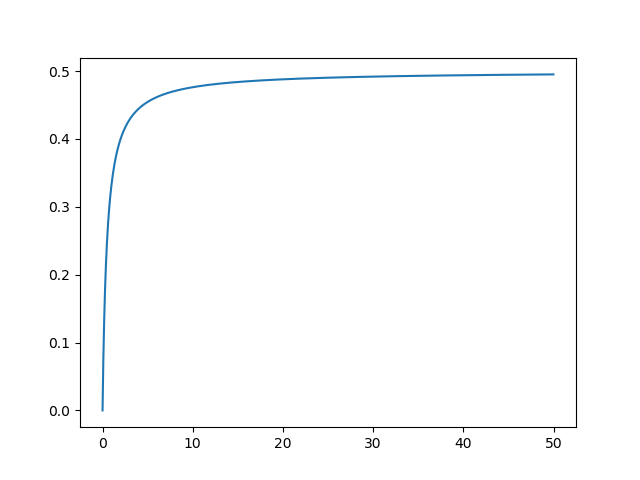


Figure 5: Graph of Threshold Voltage span when up to equal to 50

#### Example 3:

With find and which give a .

## Summary of Hysteresis Resistor Options

|  |  |  |  |
| --- | --- | --- | --- |
| Configuration | Voltage Threshold Equations | Limits of | Attributes |
|  |  |  | Setting all hysteresis resistors equal will produce threshold voltages at and |
|  |  |  | Setting will produce Threshold voltages centered around . The sum of the threshold voltages will equal 1, meaning the threshold voltages should be centered balanced.  As becomes large the threshold voltages converge at .  As becomes smaller than and the threshold voltage span becomes larger. |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | Setting allows for a shifting center point which shifts upward as becomes larger than and . |
|  |  |  | Setting allows for a shifting center point which shifts downward as becomes larger than and . |

# RC Circuit Timing

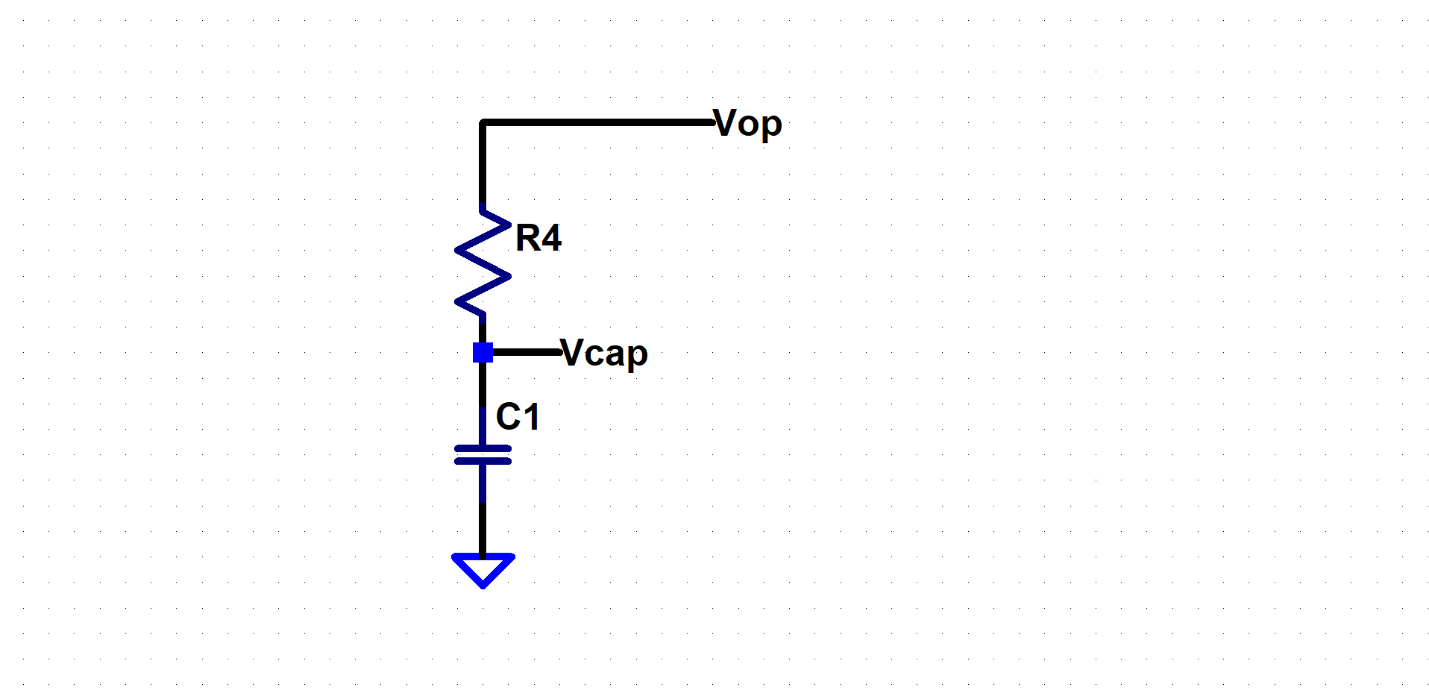


Figure 6: RC Circuit for Hysteresis Oscillator Timing

The timing for the hysteresis oscillator is controlled by a simple series RC circuit. The RC circuit charges the voltage on the cap, , to the level, when the comparator is triggered to the high state, reaching the voltage triggers the comparator to its low state where the cap is discharged to the voltage, and the cycle repeats. This set of charge and discharge cycles creates a predictable oscillation frequency.

## Generalized Equations

Using nodal analysis an equation for the RC charge and discharge voltage characteristics can be derived:

At this point in the derivation of the charge and discharge voltage characteristics, an equation for the time duration of the discharging or charging of to the desired can be found:

Assuming is 0:

Equation 14: Time Duration of a charge or discharge cycle of

Continuing the derivation of the voltage characteristics of :

Assuming is 0:

Equation 15: Voltage Characteristics over time Equation

### Simulating the Hysteresis Oscillator RC Response

The hysteresis oscillator’s RC response can be evaluated through use of a piecewise equation and the specific timing of each event the RC components experience.

#### Circuit Initialization

The first event the RC components experience is initial circuit turn on. At turn on the comparator should begin in its high state, assuming the capacitor in the RC section is fully discharged. The capacitor in the RC section would have to charge to the point that it enters the charging cycle of the periodic charge and discharge events the RC section is experiences during normal operation. The charge cycle would begin when initially hits .

The timing for this event can be determined by assuming an ideal opamp output, and using equation 14:

Equation 16: startup time for the RC section of the hysteresis oscillator to enter normal operation

With the timing known for the initial startup an equation describing the voltage characteristics at startup can be derived:

Where is equal to 0:

Equation 17: Voltage response at circuit initialization

#### Charge Event

The next event the hysteresis oscillator experiences is the first charge cycle, this event is periodic.

Because we only want to know the timing associated with the charging cycle is set to 0:

Equation 18: is the duration of a charge cycle on the RC section of the hysteresis oscillator

Using the charge timing a description of the charging voltage can be produced:

Where is defined with:

is effectively the current charge discharge cycle and begins at 0.

is defined as the discharge period, it will be considered in the next section.

is the current time value

, for , is defined using the initialization time of the specific charge cycle:

Equation 19: Voltage response during a charge cycle if circuit initialization is considered

#### Discharge Event

The final type of event that the RC section of the hysteresis oscillator experiences during normal operation is the discharge event, this is also a periodic event as the charging event was.

Because we only want to know the timing associated with the discharging cycle is set to 0:

Equation 20: is the duration of a discharge cycle on the RC section of the hysteresis oscillator

Using the discharge timing a description of the discharging voltage can be produced:

Where is again defined with:

is effectively the current charge discharge cycle and begins at 0.

is defined using the discharge cycle starting time:

Equation 21: Voltage response during a discharge cycle if circuit initialization is considered

#### Piecewise Equation Defining the RC Section Voltage Response of the Hysteresis Oscillator

Using the above voltage response equation, a piecewise function can be created to define the generalized voltage operation of the RC section of the hysteresis oscillator:

Equation 22: Piecewise function defining the voltage operation of the RC section hysteresis oscillator

Where is again defined with:

is effectively the current charge discharge cycle and begins at 0.

The specific timing duration are defined as:

#### Example 4:

Given:

Using the above sections equations for timing:

Initialization time[[2]](#footnote-2):

Charge time:

Discharge time:

Using Equation 22, the general piecewise voltage response function, and plugging in for the given values:

Where is again defined with:

Which can simplified to:

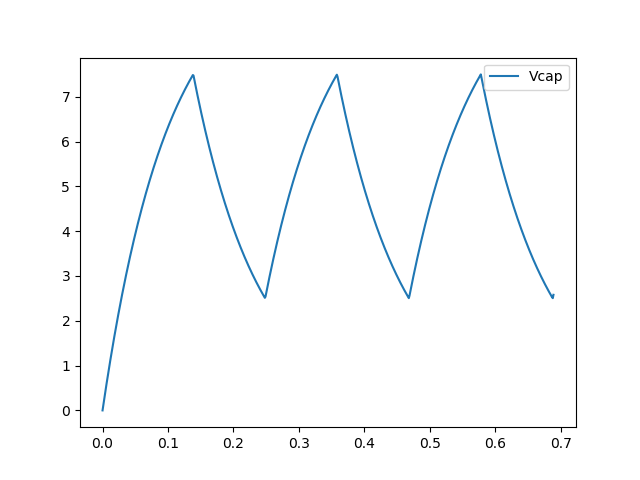


Figure 7: from circuit initialization to the end of cycle at roughly 690ms

## Duty Cycle

The timing, duration of charge and discharge, of the RC section of the hysteresis oscillator is most heavily controlled by the values of and , but the duty cycle of the square wave generated on the output pin of the opamp is almost entirely controlled by the selected hysteresis resistors. This section of this paper explores several ways to achieve a specific duty cycled square wave output from the hysteresis oscillator, and how to use circuit modifications with a known initial duty cycle to create a specific duty cycle while maintaining a specific set of trigger points, and .

This section uses the ideal opamp model contrived equations from the hysteresis resistors section of this article. This section is used to simplify hysteresis resistor selection to achieve specific duty cycles for the hysteresis oscillator output.

To evaluate more exact values evaluate and specific to the selected opamp and hysteresis resistors selected, as discussed in the introductory paragraphs at the beginning of the hysteresis resistors section, and use those evaluated values with the generalized equations for timing from the above section.

### Equal to Equal to

To evaluate the duty cycle associated with a hysteresis resistor selection consisting of equal to the charge and discharge event durations must again be found.

Let’s first examine the charge time:

Assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 23: charge time

Next the discharge time associated with setting all hysteresis resistors equal:

Assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 24: discharge time

During the charge time the output of the hysteresis oscillator is in its high state, and during the discharge time the hysteresis oscillator output is in its low state, as such these times can be used to determine a duty cycle for the hysteresis oscillator output.

Using the equations for charge and discharge time the duty cycle for can be evaluated as:

Equation 25: Duty Cycle Equation for the hysteresis oscillator using the RC section charge and discharge times

Equation 26: Duty Cycle when hysteresis resistors are all equal

This result may have seemed obvious given the equal charge and discharge times, but setting the hysteresis resistors to be equal will produce a duty cycle for the hysteresis oscillator output.

The frequency of the square wave output is found by summing the charge and discharge times:

Equation 27: Total time to complete one square wave cycle of the hysteresis oscillator

And a frequency is derived by taking the reciprocal of the total time:

Equation 28: Frequency of square wave output

Setting equal to equal to produces a RC timing that causes a 50% duty cycle, and a charge and discharge time equal to . This makes for a very predictable timing setup.

#### Example 5:

This example demonstrates the timing equations developed in the section above:

Using equation 27 the total time per cycle is:

Using equation 28 produces the output frequency:

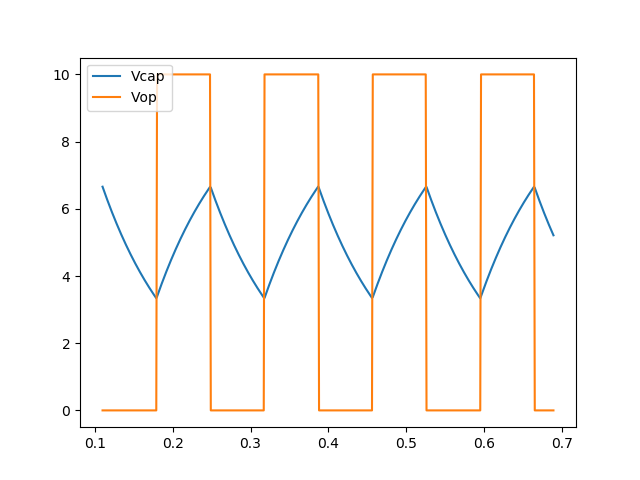


Figure : Vcc 10V; ; ; Start time 0.11s; Stop time 0.69s; ;

### Equal to

To evaluate the duty cycle associated with a hysteresis resistor selection consisting of equal to the charge and discharge event durations must be found.

Let’s first examine the charge time:

Assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 29: charge time

Keep in mind:

Next the discharge time:

Again assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 30: discharge time

Keep in mind:

Using the equation 25 the duty cycle for can be evaluated as:

Equation 31: Duty Cycle when hysteresis resistors and are equal

equal to produces a 50% duty cycle similar to setting .

#### Effect of on RC Timing

The timing of the RC section when using is effected by the choice of in that the discharge and charge time is , where is .

##### Large in Relation to

As becomes large compared to , also becomes large. Taking the limit of as it approaches infinity, as was done in the hysteresis resistors section, characterizes the trend large values of has on the timing of the RC section of the hysteresis oscillator:

As becomes large the RC timing for charging and discharging approaches 0 seconds. This makes sense because, recall, that as became large in the hysteresis resistors section the threshold voltages converged to , as such with zero transition voltage span there would be zero time required to reach the next trigger voltage as the opamp transitioned states.

##### Small in Relation to

As becomes small compared to , also becomes small. Taking the limit of as it approaches zero characterizes the trend small values of have on the timing of the RC section of the circuit:

causes an undefined result. Examining a graph of how effects the natural log term shows:

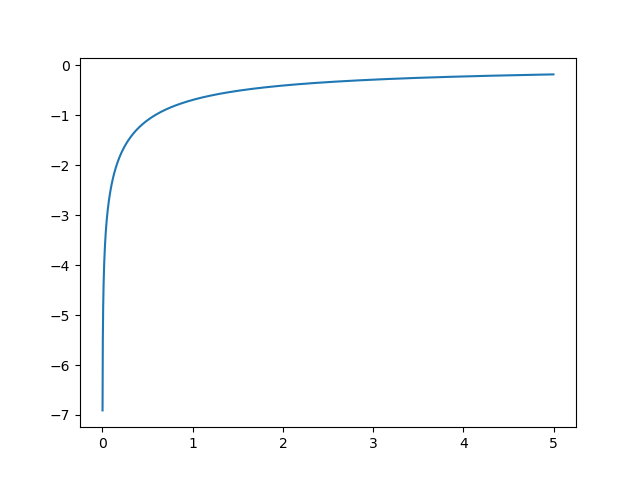


Figure : Graph of ln values as changes from 1/1000 to 5;

The above graph shows that as becomes small the overall duration of the charge and discharge timing trends longer. This results makes sense because as trended small in the hysteresis resistors section the threshold span became larger requiring more time to transition from one trigger state to another.

#### Example 6:

Example 6 builds off example 1, in that it uses the results from that section:

The results from example 1 give us:

Applying these results, main :

Given and are:

Using equation 27 the total time per cycle is:

Using equation 28 produces the output frequency:

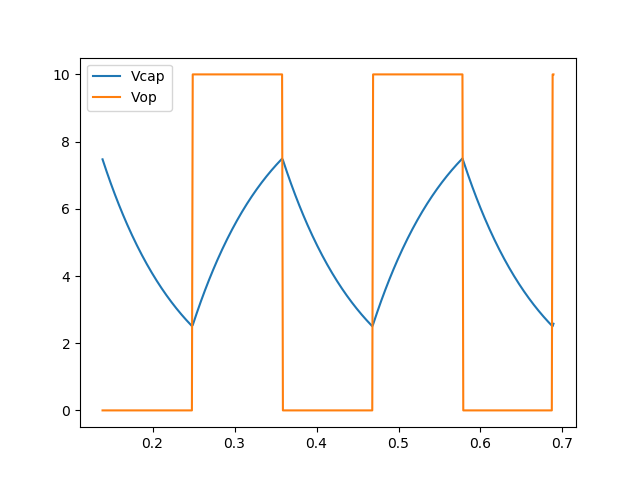


Figure : Vcc 10V; ; ; Start time 0.139s; Stop time 0.69s; ;

### Equal to

To evaluate the duty cycle associated with a hysteresis resistor selection consisting of equal to the charge and discharge event durations must be found.

The charge time:

Assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 32: charge time

Keep in mind:

Next the discharge time:

Again assuming ideal outputs for the opamp, meaning can achieve or 0V:

Equation 33: discharge time

Keep in mind:

Using the equation 25 the duty cycle for can be evaluated as:

Equation 31: Duty Cycle when hysteresis resistors and are equal

equal to produces a variable duty cycle.

#### Effect of on Duty Cycle and Timing

The timing and duty cycle of the RC section when using is effected by the choice of in that the charge time is , where is .

##### Large in Relation to

As becomes large compared to , also becomes large. Taking the limit of as it approaches infinity characterizes the trend large values of has on the timing and duty cycle of the RC section of the hysteresis oscillator:

Looking at a graph of how effects the value of the natural log term shows:

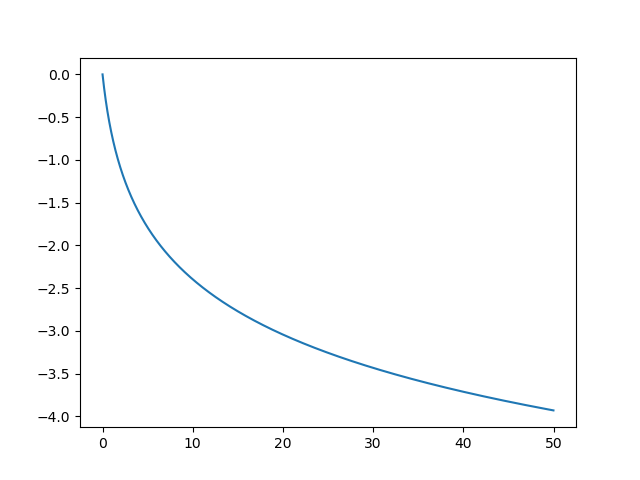


Figure : Graph of ln values as changes from 1/100 to 50;

As becomes large the value of the natural log term increases causing an overall increase in the RC section charge duration.

### Equal to

### Diode Steering

#### Fixed Diode Steering

#### PWM Control

1. This is not actually the case, as an opamp when acting as a comparator outputting its high state will generally approach but will fall short of , this is likewise for the low state of the opamp comparator, it will approach ground, 0V, but fall short of it. [↑](#footnote-ref-1)
2. Resistance time Capacitance, ohms times farads, is equal to time. R times C equals the RC time constant, which is the time it would take for a capacitor to charge to of the voltage applied to the series RC circuit [↑](#footnote-ref-2)